

A COURSE IN PROBABILISTIC COMBINATORICS

SPRING QUARTER 2020, TEN WEEKS, GRADUATE COURSE
ABOUT 40⁺ STUDENTS (+ SEVERAL FACULTY) FROM

STATISTICS

ELECTRICAL ENGINEERING

MATHEMATICS

OPERATIONS RESEARCH

COMPUTER SCIENCE PHYSICS

MEETS TWICE WEEKLY (1 1/2 HRS EACH)

COURSEWORK: TWO 10-15 PAGES PAPERS, CHOSEN FROM A LIST.

Spring Course Announcement

Stanford University Statistics Department
Sequoia Hall, 390 Jane Stanford Way
Stanford, CA 94305-4065
<http://statistics.stanford.edu/>

Probabilistic Combinatorics

MATH 233C	Live Online Tuesday, Thursday	10:30 – 11:50am
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Instructor: Persi Diaconis

This is a new graduate-level course on the interaction between probability and combinatorics, using probabilistic tools to ask what a "typical" combinatorial object looks like and using combinatorial tools (particularly generating functions) to prove theorems in probability theory. The objects will be classical things like permutations, partitions, set partitions, compositions, graphs, trees, random matrices, primes. Emphasis is on limit theorems giving "numbers" (with error terms) for sets of real-world interest. Course work will consist of two papers (of reasonable-length). Topics will include

- First examples: What is the subject about (length of longest cycle)?
- Method of conditioned limit theorems (Bayes theorem and Lecams method)
- Set partitions and coupling: From theorems to algorithms (and back)
- Graph limit theory (exponential random graph models)
- More graph limit theory (deFinetti's theorem in higher dimensions)
- Flag algebras (application to extremal graph theory)
- Permutons (and "anyons")
- Logic and combinatorics: What is a family of combinatorial objects?

The anticipated audience will have varied experience in this material, so lectures will incorporate fairly full details and subject-matter background. There are many connections between the mathematics and real-world problems, and Professor Diaconis will try to tell these stories.

LECTURES

(1) INTRO: METRICS ON PERMUTATIONS, CENTRAL LIMIT THEOREMS

FOR INVERSIONS, # CYCLES, DESCAISE BUIK-BIIFT-JOHANSSON,
EDGEBURGH CORRECTORS, EXACT COMPUTATION, MCMC

(2) CYCLES IN PERMUTATIONS (SHARP-LLOYD PAPER) LONGEST, SMALLEST, POISSONIZATION, MOMENT THEORY, REALITY CHECK (HOW LARGE DOES n HAVE TO BE?)

$$C_n(y_1, \dots, y_n) = \frac{1}{n!} \sum_{\sigma \in S_n} \prod_{i=1}^{a_i(\sigma)} y_i^{a_i(\sigma)}$$

$$C(t) = \sum_{n=0}^{\infty} C_n t^n = e^{\sum_{i=1}^{\infty} \frac{y_i t^i}{i}}$$

$\Rightarrow \{a_i(\sigma)\}$ ARE INDEPENDENT POISSON(y_i)

\Rightarrow LONG CYCLES GIVEN BY STICK BREAKING (POISSON-DILICHET)

(3) TAIGERIAN THEOREMS FOR ASYMPTOTICS OF COEFFICIENTS OF GENERATING FUNCTIONS

GENERALIZATIONS OF CYCLE INDEX TO FINITE GROUPS OF LIE TYPE

' PICK $m \in GL_n(F_q)$ AT RANDOM, HOW MANY

JORDAN BLOCKS DOES IT HAVE, HOW BIG, ...

(J. FULMAN, RANDOM MATRIX THEORY OVER FINITE FIELDS)

(4) MEASURES FOR MEASURES APPLICATIONS OF CYCLE THEORY TO

DIRICHLET RANDOM MEASURES. BETA AND DIRICHLET DISTRIBUTION

$P(\chi), D_\alpha \in P(P(\chi))$

' PICK $F \in P(\chi)$ FROM D_α , WHAT'S DISTRIBUTION OF
 $\int_X f(\chi) F(d\chi)$?

(5) CONDITIONAL LIMIT THEOREMS (POISSONIZATION) FOR

- BALLS IN BOXES
- COMPOSITIONS
- SAMPLING WITHOUT REPLACEMENT

WHEN CAN YOU RANDOMIZE A PARAMETER AND MAKE COMPONENTS INDEPENDENT

HOW DO YOU 'DERANDOMIZE' (GO BACK),

(6) PARTITIONS OF n $X_k(\gamma) = \# \text{PARTS OF } \gamma \text{ OF SIZE } k$

$$(\text{small parts}) \quad P_n\left(\frac{\pi}{\sqrt{n}} h X_k \leq x\right) \rightarrow 1 - e^{-x} \quad (\text{+ ALL INDEPENDENT})$$

(LARGE PARTS) LET $\gamma_t, t=1, 2, \dots$ BE x^{th} LARGEST PART

$$\lim_n P_n\left(\frac{\pi}{\sqrt{n}} \gamma_t - \log \frac{\sqrt{n}}{\pi} \leq x\right) = e^{-e^{-x}}$$

AND STICK BREAKING DESCRIPTION OF $\{\gamma_t\}$. ALL FOLLOW FROM

$$\prod_{i=1}^{\infty} (1 - f_i)^{-1} = \sum_{n=0}^{\infty} P(n) f^n$$

(7) BiRDAYS, COVERAGE PROBLEMS AND POISSON CLUMPING HEURISTIC

(8) CONDITIONAL LIMIT THEOREMS II (LEMIRE'S METHOD)

LET X_1, X_2, \dots, X_n BE iid, $T_n(X_1, \dots, X_n) \in \mathbb{R}^d$

WHAT TO STUDY $U_n = U_n(X_1, \dots, X_n) \mid T_n = t_n$

IDEA IF $\left(\frac{T_n}{n}\right) \xrightarrow{D} \left(\frac{T}{n}\right)$, THEN (CONDITIONS) $\mathcal{L}(U_n \mid T_n = t_n) \xrightarrow{D} \mathcal{L}(U \mid T = t)$

(9) EQUIVALENCE OF ENSEMBLES, DEFINOTTI'S THEOREM, CONDITIONS IN CORNER

(10) FROM ALGORITHM TO THEOREM (SEE THE ATTACHED FILE)

(11) Haar measure How to understand a random matrix

(see my article 'PATTERNS IN EIGENVALUES' Bull. AMS.)

More illustrations of 'From Algorithm to Theorem'

(12) GRAPH LIMIT THEORY I

REALITY THEORY TO GET AWAY from $G(\lambda, p)$

(KNUTH-STANFORD GRAPH BASE)

GRAPH Homomorphisms, CONVERGENCE, CUT DISTANCE, STATEMENT OF
MAIN THEOREMS: CONVERGENCE $\Leftrightarrow \exists W$ (GRAPH LIMIT)

COMPACTNESS OF {GRAPH LIMITS}

EXAMPLES ($G(\lambda, p)$, BLOCK MODELS) GRAPHS WITH WEIGHTS

CONNECTIONS TO DEFINOTTI'S THEOREM (COCOON-HOOVEN), PSYCHOLOGY OF VISION

(13) GRAPH LIMIT THEORY II

CLASSIC RANDOM GRAPHS, (+ JUMPS AND PSEUDO RANDOM)

(14) GRAPH LIMIT THEORY III PROOF OF COMPACTNESS (VIA MAPPINGS)

(15) PERMUTATION LIMIT THEORY Definitions, main theorems, examples

APPLICATIONS IN STATISTICS

16 THE RANDOM GRAPH (Following PETER CAMERON'S SURVEY)

(17) NON UNIFORM DISTRIBUTION, PATIENCE SORTING AND THE BOIX-DEIFF-JOHANSSON THEOREM (LONGEST ↗ SUBSEQUENCE)

(18, 19) FLAG ALGEBRAS AND ANYONS LEO COLEGLIA WO SPOKE

(20) LAST LECTURE. OPEN PROBLEMS, REVIEW, TOPICS NOT COVERED

. For σ uniform in S_n , $F(\sigma) = \# \text{fixed pts}$, $F(6) \Rightarrow \text{Poisson}(6)$

$$+ E(F(\sigma)^k) = \sum_{\sigma} \frac{6^k}{k!} \quad 0 \leq k \leq n.$$

SAME FOR ALL COKERIAN GROUPS

• CLASSICAL COMPACT LIE GROUPS

$$\int_{\Omega_n} t_n(m)^k dm = \int_0^\infty r^k \frac{e^{-r^2/4}}{\sqrt{\pi n}} dr \quad 0 \leq k \leq 2n+1$$

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35 KIDS TOOK THE COURSE FOR CREDIT, 3 OF THE FINAL PROJECTS BECAME PUBLISHED PAPERS (SO FAR).